

QUIZ 3: LESSON 1B
JANUARY 14, 2019

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [3 pts] Evaluate $\int 12x(x+1)^2 dx$.

$$\begin{aligned}
 u &= x+1 \Rightarrow x = u-1 & \int 12x(x+1)^2 dx \\
 \frac{du}{dx} &= 1 \Rightarrow dx = du & = \int 12(u-1)u^2 du \\
 & & = \int 12(u^3 - u^2) du
 \end{aligned}$$

$$\begin{aligned}
 &= 12 \left(\frac{1}{4} u^4 - \frac{1}{3} u^3 \right) + C \\
 &= 3u^4 - 4u^3 + C \\
 &= \boxed{3(x+1)^4 - 4(x+1)^3 + C}
 \end{aligned}$$

2. [7 pts] If the area of the region under the curve

$$y = \frac{1}{\sqrt{5x+2}}$$

over the interval $0 \leq x \leq a$ is 8, then what is a ?

$$8 = \int_0^a \frac{1}{\sqrt{5x+2}} dx = \int_0^a (5x+2)^{-1/2} dx$$

$$\begin{aligned}
 \boxed{u &= 5x+2} \\
 \frac{du}{dx} &= 5 \\
 \Rightarrow dx &= \frac{du}{5}
 \end{aligned}$$

$$= \int_{u(0)}^{u(a)} u^{-1/2} \frac{du}{5}$$

$$\begin{aligned}
 &= \left(\frac{1}{5} \right) \left(\frac{1}{-1/2+1} \right) u^{-1/2+1} \Big|_{u(0)}^{u(a)} \\
 &= \left(\frac{1}{5} \right) \left(\frac{1}{1/2} \right) u^{1/2} \Big|_{u(0)}^{u(a)} \\
 &= \frac{2}{5} u^{1/2} \Big|_{u(0)}^{u(a)} \\
 &= \frac{2}{5} (5x+2)^{1/2} \Big|_0^a \\
 &= \frac{2}{5} (5a+2)^{1/2} - \frac{2}{5} (5(0)+2)^{1/2} \\
 &= \frac{2}{5} (5a+2)^{1/2} - \frac{2}{5} \sqrt{2}
 \end{aligned}$$

So,

$$8 = \frac{2}{5} (5a+2)^{1/2} - \frac{2}{5} \sqrt{2}$$

$$\frac{5}{2} (8) = (5a+2)^{1/2} - \sqrt{2}$$

$$20 + \sqrt{2} = (5a+2)^{1/2}$$

$$(20 + \sqrt{2})^2 = 5a+2$$

$$402 + 40\sqrt{2}$$

$$400 + 40\sqrt{2} = 5a$$

$$\boxed{80 + 8\sqrt{2}} = a$$